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Statistics-based method for determination of drag coefficient for nonlinear porous flow in calcareous sand soil

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Abstract
Generally, drag force applied by soil particles to seeping pore water is described as a type of body force in the form of hydraulic gradient I. For steady porous flow, the hydraulic gradient was widely formulated by the Forchheimer equation, containing two Forchheimer drag coefficients a and b. In this study, a simple and novel experimental device is designated to study the seepage characteristics of pore water in soils. Taking the calcareous sand soil coming from the South China Sea (SCS) as a typical porous medium, three parallel tests of seepage flow are performed. Based on the experimental data of apparent speed u of pore water and corresponding hydraulic gradient I, a statistic-based methodology is proposed to determine drag coefficients a, b of SCS calcareous sand soil. The priority of the proposed methodology is that the statistic distribution of measured parameters can be clearly observed through a small number of tests. Comparative study shows that the drag coefficient a and b of SCS calcareous sand soil determined by the statistic-based method proposed in this study can perfectly describe the corresponding experimental data of u-I relationship. It is indicated that the proposed statistics-based methodology to determine drag coefficient of soil is reliable and feasible.

Keywords South China Sea · Calcareous sand soil · Nonlinear porous flow · Forchheimer drag coefficient · Statistical theory · Coral reef

Introduction

Soil is an aggregation of a great number of particles with various scales. There is an interconnected network of void in soils, making fluids such as pore water and air can flow through it. Permeability is an important physical property of soils. It determines the speed of soil consolidation, as well as the prone degree of liquefaction of soil under cyclic loading, such as ocean or seismic waves.

The seepage capacity of soils generally is characterized by the permeability coefficient k in soil mechanics. There are two typical methods, constant water head method and variable water head method, which are widely suggested for the measurement of permeability coefficient k of soils. In addition to the permeability coefficient, there is another important physical parameter referred as drag force, which can be used to characterize the seepage capacity of soils. Drag force is a type of resistance force between water and soil particles when pore water flows in the void of soils. Drag force is attributed to two aspects of contribution in physical mechanism. The first contribution is the viscous shear stress in the boundary layers near to outer surface of soil particles. This contribution to drag force cannot be ignored, and even seepage speed of pore water is small (Darcy flow). However, drag force also plays an important role when seepage speed of pore water is great (non-Darcy flow). Even though the viscous shear stress in boundary layers is tiny for each soil particle, its contribution to drag force is considerable for in an aggregation of a great number of soil particles. The second contribution is the interaction force between pore water and soil particles due to the momentum change of pore water (inertial effect) when flowing around soil particles. This contribution to drag force is always there in both the cases of low and high flowing speed. As a whole, viscous shear stress is dominant when seepage speed of pore water is in the Darcy flow regime compared with inertial effect. However, inertial effects due to the momentum change of pore water become dominant rather than viscous effect in the post-Darcy regimes corresponding to relatively great seepage speed.

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Drag force applied by soil particles to pore water of course is opposite to the flowing direction of pore water. Undoubtedly, drag force will dissipate the kinetic energy of pore water or generate a hydraulic gradient. For a single smooth spherical soil particle, there has been an analytical solution for the drag force applied to flowing water (Anderson 1994; Fair et al. 1968; Lin and Karunarathna 2007). However, for an aggregation of a great number of real soil particles with irregular shapes, an analytical solution is not available due to the complexity of the analytical process. The only feasible way is to describe the drag force macroscopically adopting the concept of volume average (Hsu et al. 2002; Liu et al. 1999) in mathematics. As a result, drag force applied to pore water in soils is generally formulated as a type of body force in the form of hydraulic gradient I. For low speed linear seepage flow, Darcy (1856) firstly proposed $I = au$ to describe the hydraulic gradient induced by drag force, where u is the apparent speed of pore water, a is a coefficient. For nonlinear seepage flow, Forchheimer (1901) proposed $I = au + bu^2$ to describe the drag force induced hydraulic gradient in soil, where b is also a coefficient. For high speed turbulent flow, Polubarinova-Kochina (Polubarinova-Kocina 1952) further proposed $I = au + bu^2 + c\partial u/\partial t$ to consider the effect of inertial term. However, Van Gent (1995) concluded that the effect of inertial term is not significant. It was suggested that the coefficient c can be determined as 0.34 in computation (Van Gent 1995; Hsu et al. 2002; Higuera et al. 2014). a, b and c generally are referred to as Forchheimer drag coefficients of soils. They are mainly related to the porosity n of soils, mean size d_{50} and shape of soil particles.

So far, there have been various empirical formulations available to estimate the value of drag coefficients a, b of soils; see Ergun 1952; Engelund 1953; Van Gent 1995; Lin and Karunarathna 2007. However, the most widely used at present is formulated as the following (Engelund 1953; Liu et al. 1999; Higuera et al. 2014):

$$a = \alpha \frac{(1-n)^2}{n^3} \frac{\upsilon}{g d_{50}^2} \tag{1}$$

$$b = \beta \frac{1-n}{n^3} \frac{1}{g d_{50}} \tag{2}$$

in which $\nu = 1.002 \times 10^{-3}$ Pa•s is the dynamic viscosity of pore water, n is porosity, d_{50} is the mean size of soil particles. α and β are two correction coefficients to consider the effect of soil particle shape factors. The intent of establishing an empirical formulation was to conveniently and quickly estimate drag coefficients a, b of soils. However, the reliability of estimation using empirical formulations is difficult to guarantee sometimes. The most reliable way in any circumstance is to perform experimental tests to determine the drag coefficients a, b of soils. In previous literature, few works had been conducted to experimentally measure the drag coefficient of soils or fractured rock mass; see Pradeep Kumar et al. 2004; Sidiropoulou et al. 2007; Chukwudozie et al. 2012; Chen et al. 2015; however, their experimental devices generally were a little complicated in structures. Most recently, Wang et al. (2018) proposes a photogrammetry-based method to measure the drag force coefficient of a single calcareous sand particle with irregular shape. However, this method is not applicable for an aggregation of a great number of soil particles.

In this study, a novel and simple experimental device is designed (Wang et al. 2017) to study the seepage characteristics of pore water in the calcareous sand soil from the South China Sea (SCS). A statistic-based methodology is proposed to determine drag coefficients a, b of soils based on the obtained experimental data of apparent speed u of pore water and corresponding hydraulic gradient I. The comparative study shows that drag coefficients a and b of SCS calcareous sand soil determined by the statistic-based method proposed in this study can perfectly describe the corresponding experimental data of the u-I relationship. It is indicated that the proposed statistic-based methodology to determine the drag coefficient of soil is reliable and feasible. Uncertainty is always present in all types of experiments, which results in the unreliability of test results. In this work, the statistics idea is taken into consideration when determining physical parameters. The proposed method in this study could let engineers and researchers determine the Forchheimer drag force coefficient of porous medium more reliably by taking the statistics idea into consideration in the community of soil mechanics and CFD field.

Experimental device and methodology

A simple and novel experimental device illustrated in Fig. 1 is designed (Wang et al. 2017) to determine the drag force coefficient of porous medium, such as sands and gravels. The device is made from transparent resin glass, and mainly consists of an inlet, an outlet, two glass ball sections, a sample section for containing the soil sample and a differential pressure meter, which is connected to the two ends of a hollow resin glass cylinder through two plastic pipes. The main function of the glass ball (diameter = 5 mm) sections is to reduce the turbulence of water flow before flowing into sample soil, making the flow mostly like a laminar flow.

In tests, a pump driven by a servo motor provides the device with sufficient water through the inlet part. The seepage speed of water in the soil sample can be regulated by adjusting the output power of servo motor. Only once the seepage flow of pore water becomes steady are the flowing speed and corresponding pressure difference recorded in tests. The amount of water, ΔQ, coming through the outlet part in time interval Δt (2–10 min) is weighted using an electronic balance. Then the flux of water flowing through the porous soil sample is determined as $q = \Delta Q/\Delta t$ (m^3/s). Correspondingly, the

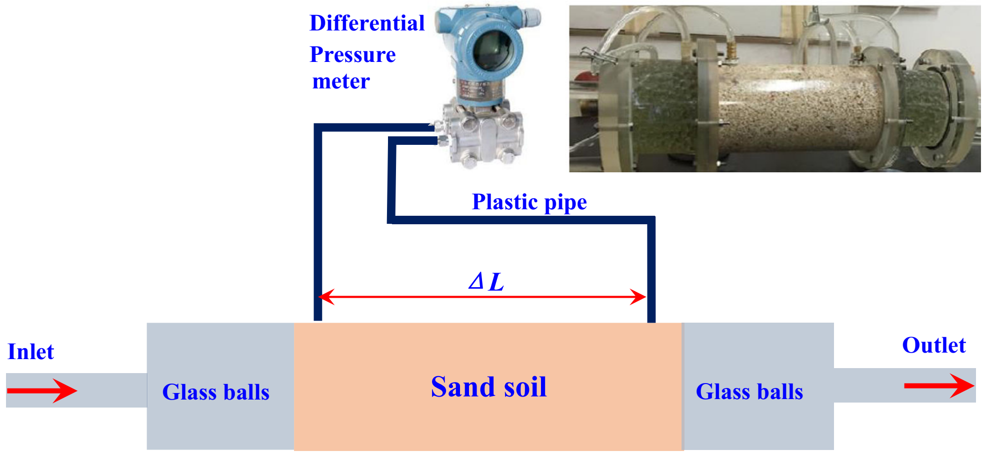


Fig. 1 Sketch of the simple experimental device for seepage flow in porous soils

seepage apparent speed of water u can be determined as $u = q/(A\rho_w g)$, where A is the net cross-sectional area of the soil sample section (diameter = 10.6 mm); ρ_w is the density of water, and g is gravity. The difference of water pressure Δp at the two ends of the soil sample is recorded by the differential pressure meter. Then the pressure gradient I is determined as $I = \Delta p/(\Delta L \rho_w g)$, where L is the distance of the two points where the two plastic pipes connected to the device (L = 185 mm). Once the seepage speed u and pressure gradient I are determined for one situation, then the permeability coefficient k in this situation can be determined by $k = u/I$, regardless of laminar flow or nonlinear flow.

According to the extended Forchheimer formulation $I = au + bu^2 + c\partial u/\partial t$, so long as three data points in u-I coordinates are measured for one soil sample, the drag force coefficients a, b and c can be estimated by solving a group of ternary quadratic equations. In this study, the effect of acceleration of seepage flow cannot be considered because only steady seepage flow in the device can be generated by the pump. Therefore, acceleration of seepage flow $\partial u/\partial t$ must be 0 in this case. The drag force coefficient a and b for steady seepage flow is the focus of this work.

For steady seepage flow, extended Forcheimer formulation is degenerated into $I = au + bu^2$. In this case, if any two data points (u_1, I_1) and (u_2, I_2) in u-I coordinates are measured for a soil sample, then the drag coefficients a and b can be determined by solving a group of binary quadratic eq. (3).

$$\begin{cases} I_1 = au_1 + bu_1^2 \\ I_2 = au_2 + bu_2^2 \end{cases} \tag{3}$$

$$\begin{pmatrix} a \\ b \end{pmatrix}^T = \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \begin{pmatrix} u_1 & u_2 \\ u_1^2 & u_2^2 \end{pmatrix}^{-1} \tag{4}$$

If a series of data points in u-I coordinates are measured, any two data points among them can be randomly picked to estimate the drag coefficients a and b according to Eq. (4). Finally, a series of drag coefficients a and b can be obtained. As a result, the statistical distribution of drag coefficients a and b are available through only one test. This type of method estimating the drag coefficient of porous medium has obvious priority because the reliability of measurement could be observed in only one test through the statistical distribution of measured parameters.

Basic properties of calcareous sand soil

The calcareous sand soil used in this study is sampled at a reclaimed island on the coral reef in the South China Sea. A real view of the sampled calcareous sand soil is shown in Fig. 2. The main chemical composition of the calcareous sand soil is $CaCO_3$ and $MgCO_3$. The gradation of soil particle size is relatively poor, as illustrated in Fig. 3. As shown in Fig. 2, there are a number of large particles in the in situ samples. They are actually the remains of coral branches. The

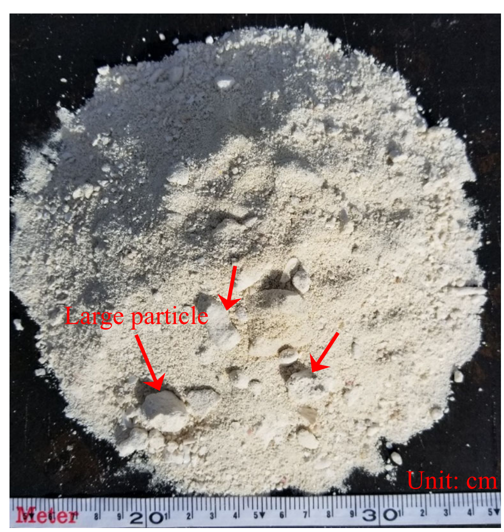


Fig. 2 A real view of calcareous sand soil sampled from the South China Sea

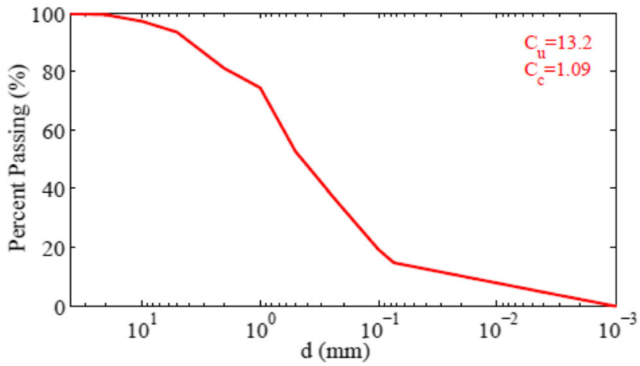


Fig. 3 Particle-size distribution curve of SCS calcareous sand soil (d_{50} = 0.5 mm)

maximum size of these large particles in the calcareous sand soil used in this study reaches 3 cm.

A series of laboratory tests have been performed to measure the basic properties of SCS calcareous sand soil. The maximum dry density $\rho_{d\ max}$ is 1.75 g/cm^3, minimum dry density $\rho_{d\ min}$ is 1.15 g/cm^3. Specific gravity G_s is in the range of 2.7 to 2.83. In the study, samples with a dry density ρ_d = 1.6 g/cm^3 are used in the three parallel tests of seepage flow for SCS calcareous sand soil. The shear strength of the samples with a dry density ρ_d = 1.6 g/cm^3 is also measured by the consolidated-undrained (CD) method on conventional triaxial equipment. The stress-strain curves and Mohr circles at peak strength under confining pressure 50 kPa, 100 kPa, 200 kPa, 400 kPa are demonstrated in Fig. 4. It is measured that the cohesion C = 100.5 kPa, and the internal friction angle φ = 44.1° for the SCS calcareous sand soil. The presence of cohesion C = 100.5 kPa is due to the existence of a number of large soil particles in the SCS calcareous sand soil. When a large soil particle is crossed by shear bands in triaxial tests, the particle strength will bring contribution to the test result of cohesion of soil. Actually, there is no cohesion between soil particles for sand soil. This tested cohesion of soils is only an apparent cohesion. The test result of cohesion for sand soil being non-zero is a widely existing phenomenon if there are a number of large soil particles contained. Because it is very difficult for soil samples to become fully saturated, the saturation of soil samples is written as S_r = 1 or S_r ~1 only for approximation.

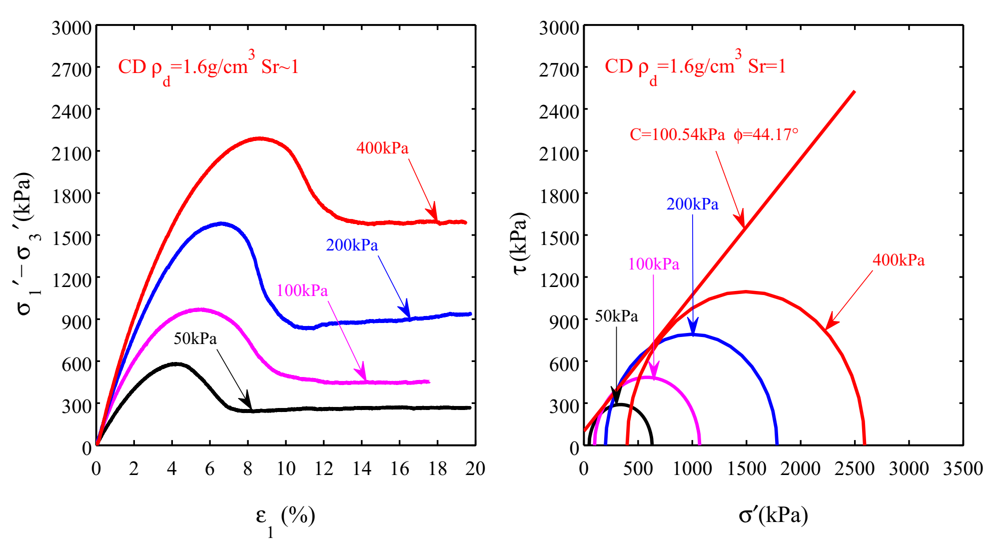
Experimental results analysis

u-I seepage relationship

It is well known that a geotechincal test result will be affected by a number of factors, such as sensor accuracy, manual operation, inconsistency between soil samples. To avoid these types of random test results, a suggested feasible way is to perform the same test several times, namely parallel tests. In this study, three parallel tests are performed to determine the drag force coefficient of calcareous sand soil sampled from the South China Sea.

The u-*I* seepage relationship of the calcareous sand soil obtained from the three parallel tests are demonstrated in Fig. 5. It can be observed that the experimental results of the *u-I* relationship obtained from the three parallel tests is indeed slightly different, indicating that it is necessary to perform parallel tests if more reliable test results are expected. In Fig. 5, it is also found that the *u-I* relationship is not linear in all ranges of hydraulic gradient *I*. When the hydraulic gradient *I* is less than a critical value I_0, for example 12.5 in parallel test #1, the *u-I* relationship basically is linear, satisfying Darcy's flow. If hydraulic gradient *I* is greater than the critical value I_0, the porous flow is no longer Darcy's flow, but nonlinear porous flow. This critical value I_0 of the hydraulic gradient at which transformation occurs from Darcy's

Fig. 4 Stress-strain curves and Mohr circles at peak strength of SCS calcareous sand soil



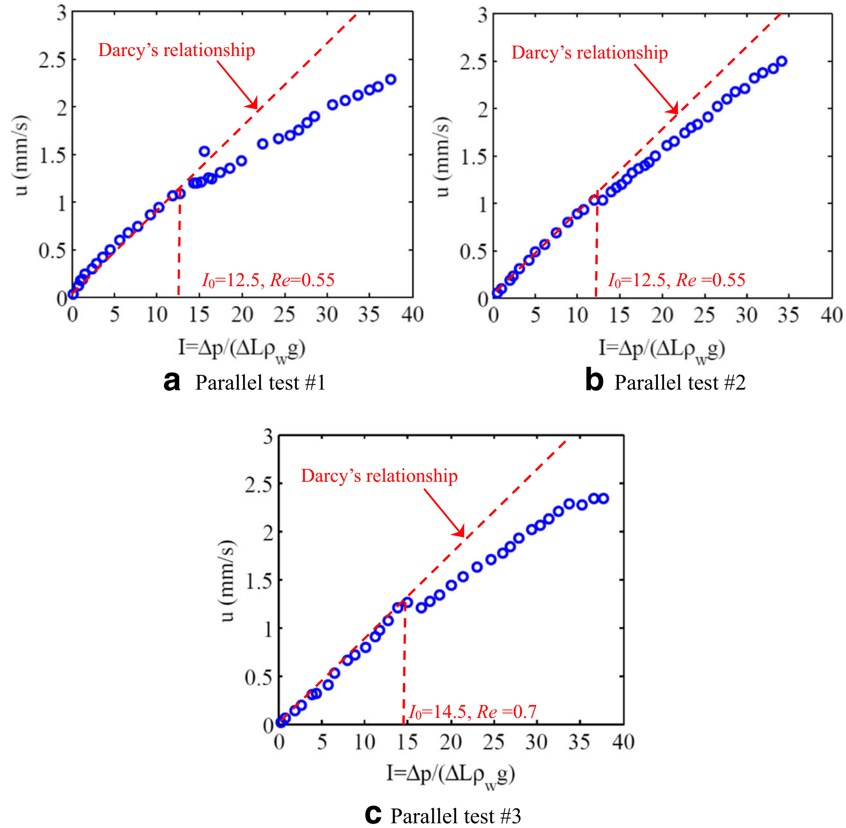


Fig. 5 Experimental *u-I* relationships in three sets of parallel tests for SCS calcareous sand soil

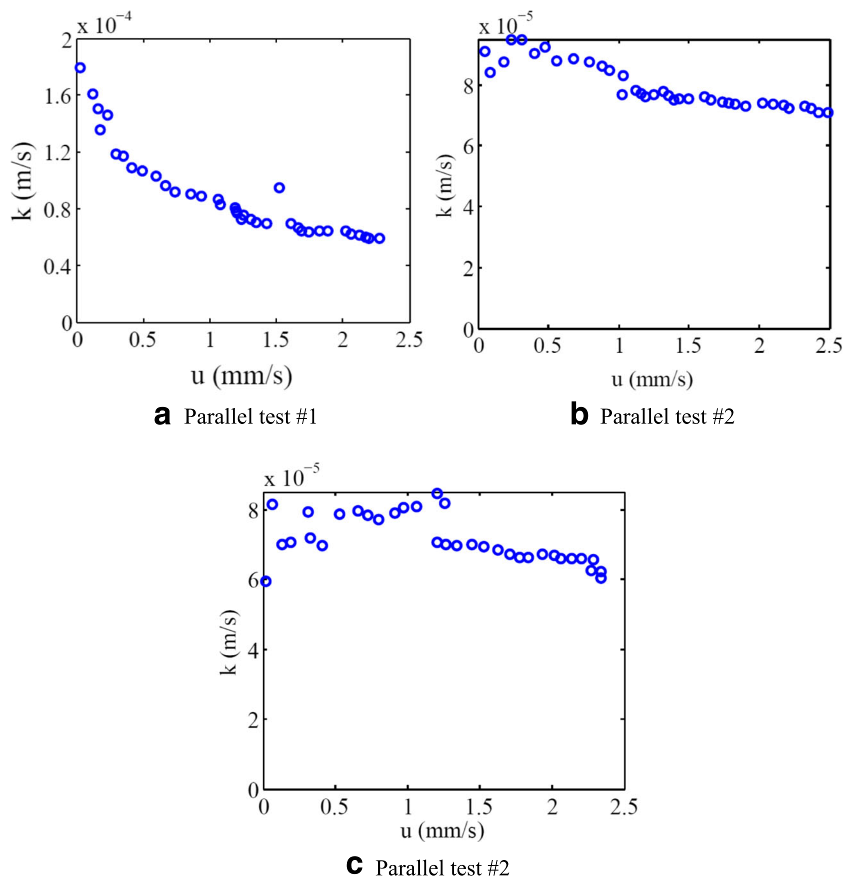


Fig. 6 Experimental *k-u* relationship in three sets of parallel test for SCS calcareous sand soil

flow to nonlinear porous flow is 12.5 in parallel tests #1 and #2, 14.5 in parallel test #3. Overall, this critical value I_0 is in the range of 12 to 15 for the calcareous sand soil sampled from the South China Sea. Correspondingly, Reynold's number $Re = \rho_w u d_{50}/\mu$ is 0.55 in parallel tests #1 and #2 and 0.7 in parallel test #3 at this transformation status. Previous studies shown that the seepage in porous medium generally was Darcy's flow when $Re \leq 1.0$. However, it is shown that the seepage still can be non-Darcy's flow when Reynold's number Re is in the range of 0.55/0.7 to 1.0, indicating that the previous conclusion on the critical Reynold's number at transformation status from Darcy's flow to non-Darcy's flow is questionable for calcareous sand soil. More investigation on this critical Reynold's number for calcareous sand soil is needed in the future.

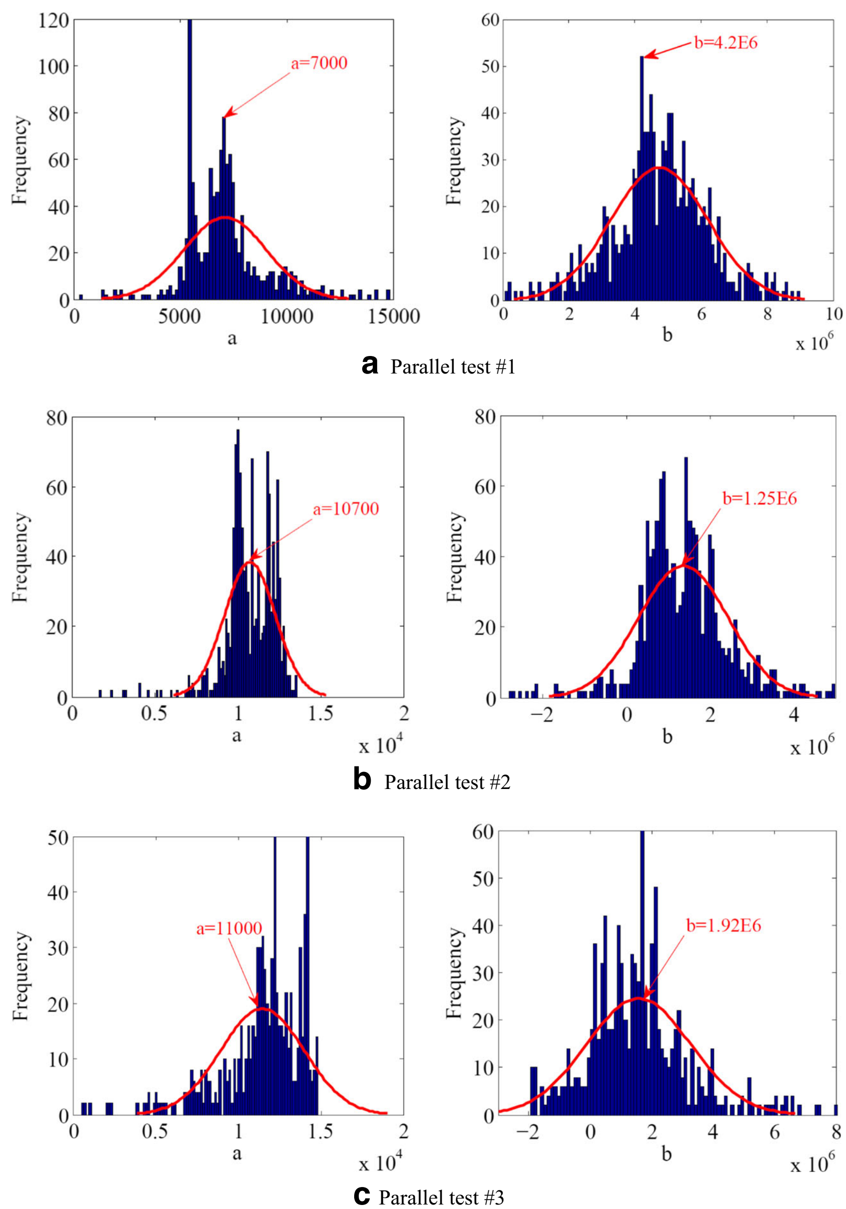
Permeability-seepage speed relationship

So long as the water flux q (m^3/s) is flowing through soil samples and the pressure difference Δp are measured in tests, the permeability coefficient k of soil can be determined as

$$k = \frac{q}{AI} = \frac{u}{I} = \frac{qL\rho_w g}{A\Delta p} \tag{5}$$

where u is the apparent speed of flowing water, A is the area of soil sample cross section. The relationship between permeability coefficient and apparent speed of seepage water for SCS calcareous sand soil are illustrated in Fig. 6. It is observed that permeability coefficient of SCS calcareous sand soil is at the magnitude of O (10^{-5}–10^{-4} m/s). The permeability

Fig. 7 Statistical distribution of the drag force coefficients a and b of SCS calcareous sand soil



coefficient measured in parallel test #1 is 0.6–1.8 × 10^{-4} m/s, 0.7–0.95 × 10^{-4} m/s in parallel test #2, and 0.6–0.85 × 10^{-4} m/s in parallel test #3. Additionally, the permeability coefficient of soil is not a constant. It decreases with increasing seepage speed u. This phenomenon can be attributed to the presence of drag force between flowing water and soil particles. This drag force is opposite to the direction of flowing water and positively related to flow speed. Great flow speed means great drag force, making pore water more difficult to seep in soil. As a result, the permeability coefficient of soil is negatively related to the flowing speed of pore water. Traditionally, only one permeability coefficient is generally given to a soil sample to characterize the ability of water seepage in the soil. Actually, this is not rigorous based on the experimental results presented here. A flow speed of pore water should be given when a permeability coefficient of soil is given.

Statistical determination of drag force coefficient

As presented previously, Forchheimer formulation $I = au + bu^2 + c\partial u/\partial t$ can be degenerated into $I = au + bu^2$ for steady seepage flow ($\partial u/\partial t = 0$). if any two data points in u-I coordinates are measured for a soil sample, then the drag force coefficients a and b can be determined by solving a group of binary quadratic equations. As shown in Fig. 5, a series of data points in u-I coordinates have been measured for SCS calcareous sand soil through three parallel tests. Any two data points of u-I measured in a parallel test can be picked to estimate the drag force coefficients a and b. As a result, a series of drag force coefficients a and b can be obtained.

The statistical distribution of the drag force coefficients a and b of SCS calcareous sand soil determined by the three parallel tests are illustrated in Fig. 7. It is found that the distribution of the drag force coefficients a and b of SCS calcareous sand soil all basically obey normal distribution. The mean value of the drag force coefficients a and b is at the magnitude of $O(10^4)$ and $O(10^6)$, respectively. Specifically, the mean of a and b is measured as 7000, 4.2 × 10^6 by the parallel test #1. They are 10,700, 1.25 × 10^6 by the parallel test #2, and 11,000, 1.92 × 10^6 by the parallel test #3. Because the statistical distribution of measured parameters can be clearly observed, the test methodology and the post-process method for experimental data proposed in this study can guarantee a high reliability of the measured drag force coefficient a and b of SCS calcareous sand soil.

After determining drag force coefficients a and b of SCS calcareous sand soil adopting statistical methodology, it is necessary to check whether these drag force coefficients a and b can reasonably describe the relationship in mathematics between apparent seeping speed u and hydraulic gradient I in the three parallel tests. Comparison between these experiment data and the mathematical description of the u-I relationship of SCS calcareous sand soil in the three parallel tests is illustrated in Fig. 8. It can be clearly observed that the mathematical description of the u-I relationships of SCS calcareous sand soil adopting the drag force coefficient a and b determined

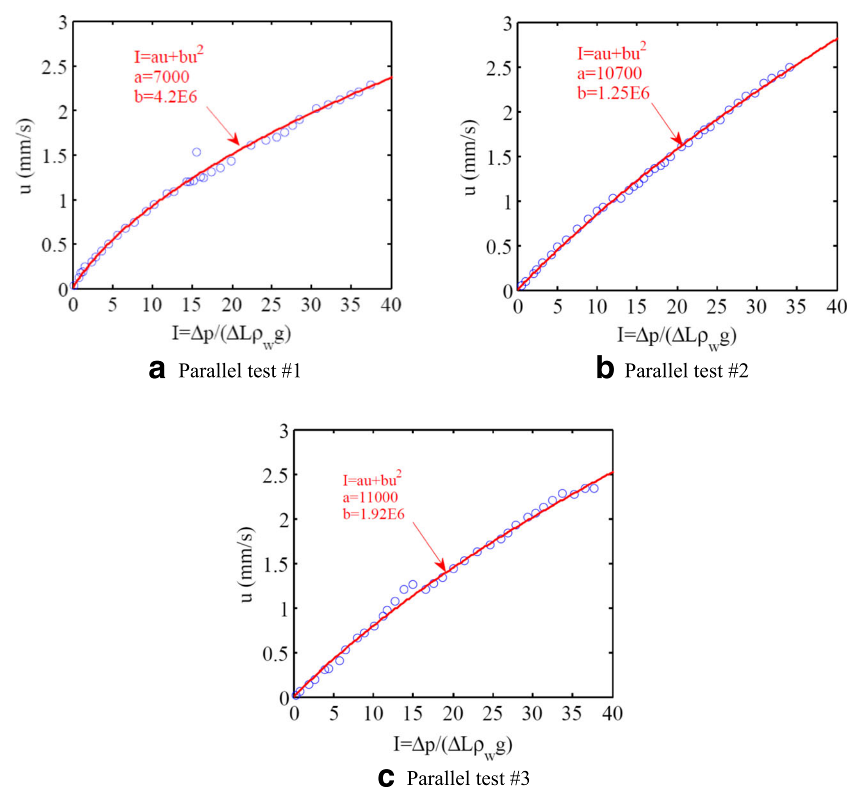


Fig. 8 Comparison between experimental data and mathematical description of the u-I relationship of SCS calcareous sand soil

by statistical methodology all basically agree perfectly with their corresponding experimental data. Figure 8 further proves that the statistical methodology of measuring drag force coefficients *a* and *b* of soil proposed in this study is reliable and feasible.

Conclusions

In this study, a simple experimental device is used to study the seepage flow characteristic of pore water in soils. The relationship between apparent speed *u* of pore water and hydraulic gradient *I*, and the permeability of soils corresponding to various apparent speed *u* of pore water can be quantitatively measured. For the calcareous sand soil from the South China Sea (SCS), three parallel tests of seepage are performed adopting the simple experimental device. Based on these test data of apparent speed *u* of pore water and hydraulic gradient *I*, a statistic-based methodology is proposed to determine the drag force coefficients *a* and *b* of SCS calcareous sand soil. An important priority of the proposed method is that the statistical distribution of drag force coefficients *a* and *b* of soil can be directly and clearly observed. As a result, the reliability of measurement can be easily assessed. The measurement results in this study illustrate that the drag force coefficients *a* and *b* of SCS calcareous sand are both obey the normal distribution. It is suggested that drag force coefficients *a* and *b* of SCS calcareous sand can be determined as the mean value of normal distribution. The comparative study shows that the drag coefficients *a* and *b* of SCS calcareous sand determined by the statistic-based method proposed in this study can perfectly describe corresponding test data of *u*-*I* relationship. It is indicated that the proposed statistic-based methodology to determine the drag force coefficient of soil is reliable and feasible. However, it is worth mentioning that the experimental device used in this study is not applicable for soils with apparently small permeability coefficient, such as clay soil and silty soil.

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